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Letter to the Editor

The meshless Galerkin method for pressure distribution simulation of horizontal well reservoir

Shuyong Hu [a,*], Junxiu Ma [a], Renyan Zhuo [a], Binyuan Qin [b]

[a] State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, China
[b] Directional Drilling Department, Xinjiang Zhengtong Industrial Limited Company, Urumqi, China

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ABSTRACT

This paper provides a novel three-dimensional meshless Galerkin for horizontal well reservoir simulation. The pressure function is approached by moving least-square method which consists of weight function, basic function and coefficient. Based on Galerkin principle and use penalty function method, the paper deduces the meshless Galerkin numerical linear equations. Cut off the pressure distribution of the horizontal section from the simulation database of horizontal well reservoir. It demonstrates that meshless Galerkin is a feasible numerical method for the horizontal well reservoir simulation. It is useful to research complex reservoir.

1. Introduction

The main methods of researching reservoir simulation include: finite difference method, finite element method or finite volume method, and these methods are all based on mesh. Mesh generation of these methods are complicated, while meshless method can effectively overcome this defect. There are dozens of meshless methods, and the most commonly used is meshless Galerkin. So some researchers adopted meshless Galerkin to research reservoir simulation, include three main models [1–3]: circle reservoir simulation, using polar coordinates to change it to radial direction, which is one-dimensional model; two-dimensional rectangle reservoir model, the defect of this kind of model is that it does not consider well processing; vertical fractured reservoir model, this kind of model is mainly used to deal with two-dimensional problems [4]. While three-dimensional model is more complicated than one-dimensional and two-dimensional models, and few researchers applied meshless Galerkin to three-dimensional reservoirs [6,7], but there are a few cases showing that meshless Galerkin is used to horizontal well model. So we apply three-dimensional meshless Galerkin to horizontal well reservoir simulation.

2. Methodology

2.1. Moving least-square method

In domain Ω, function $u(\boldsymbol{x})$ is expressed as an approximate function.

$$u(\boldsymbol{x}) = \sum_{i=1}^{m} b_i(\boldsymbol{x}) a_i(\boldsymbol{x}) = \boldsymbol{b}^T(\boldsymbol{x}) \boldsymbol{a}(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{a}(x)$ is m-dimensional coefficient vector, $\boldsymbol{b}(x)$ is m-dimensional basic function vector.

To three-dimensional problem, basic function selects quadratic complete polynomial.

$$\boldsymbol{b} = \begin{bmatrix} 1 & x & y & z & xy & yz & zx & x^2 & y^2 & z^2 \end{bmatrix}^T. \tag{2}$$

For the moving least-square method, Eq. (1) transforms to

* Corresponding author.
E-mail address: hsylylhjb@126.com (S.Y. Hu).
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$$u(\boldsymbol{x}) = \sum_{j=1}^{n} \mathbf{N}_j^T(\boldsymbol{x})\boldsymbol{u}_j, \tag{3}$$

where

$$\boldsymbol{N}_j(\boldsymbol{x}) = \sum_{i=1}^{m} \boldsymbol{b}_i^T(\boldsymbol{x})\left[\boldsymbol{A}^{-1}(\boldsymbol{x})\mathbf{B}(\boldsymbol{x})\right]_{ij}, \tag{3a}$$

$$\boldsymbol{A}(\boldsymbol{x}) = \sum_{j=1}^{n} w_j(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x}_j)\boldsymbol{b}^T(\boldsymbol{x}_j), \tag{3b}$$

$$\boldsymbol{B}(\boldsymbol{x}) = [\, w_1(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x}_1) \quad \cdots \quad w_n(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x}_n)\,], \tag{3c}$$

where $w_j(x)$ is the weighted function of the jth node at $(x, y)^T$, $\boldsymbol{N}_j$ is the shape function of the jth node.

The partial derivative of the shape function $\boldsymbol{N}_j$ is

$$\begin{aligned}\boldsymbol{N}_{j,k} = \sum_{i=1}^{m}\Big\{&\boldsymbol{b}_{i,k}\left[\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\right]_{ij} \\ &+ \boldsymbol{p}_i\left[\boldsymbol{A}_{,k}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x}) + \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}_{,k}(\boldsymbol{x})\right]_{ij}\Big\},\end{aligned} \tag{4}$$

$$\boldsymbol{A}_{,k}^{-1} = -\boldsymbol{A}^{-1}\boldsymbol{A}_{,k}\boldsymbol{A}^{-1}. \tag{5}$$

The weighted function is very important. It affects the stability and precision the numerical calculation result. It must subject to that the weighted function value is nonnegative and the function has decay property. So this paper selects Gaussian weight function.

$$w(r) = \begin{cases} \dfrac{e^{-(r/d)^2c^2} - e^{-c^2}}{1 - e^{-c^2}} & r \le 1 \\ 0 & r > 1 \end{cases}, \tag{6}$$

where r is the distance between $\boldsymbol{x}$ and $\boldsymbol{x}_i$; c is a small positive; d is the affected radius.

2.2. Meshless Galerkin

Darcy's law is

$$\boldsymbol{v} = -\frac{\boldsymbol{k}}{\mu}\cdot gradp, \tag{7}$$

where, $\boldsymbol{v}$ is seepage velocity, μm/s; grad p is pressure gradient, MPa/m; μ is fluid viscosity, MPa·s; $\boldsymbol{k}$ is permeability tensor, mD.

Assume that the gravitational effect and starting pressure gradient are ignored, and the permeability of the formation is isotropic, then the differential equation of three-dimension single phase steady state flow is

$$div\left(\frac{k}{\mu}gradp\right) = 0, \tag{8}$$

where k is the permeability of the formation is isotropic, mD.

Component form of the Eq. (8) is

$$-\frac{k}{\mu}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) = 0. \tag{9}$$

The Dirichlet boundary condition of the Eq. (9) is

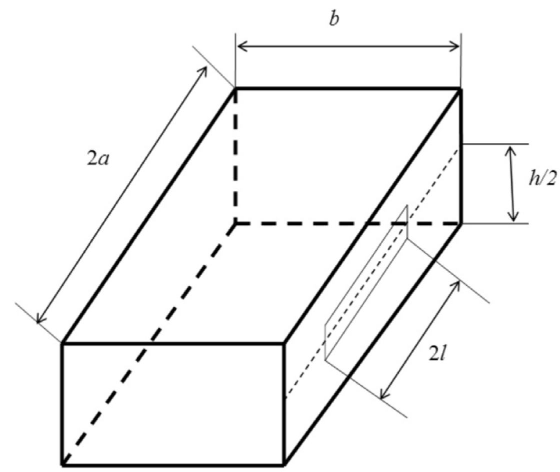

Fig. 1. Vertical section diagram of a horizontal well.

$$p|_{\Gamma_1} = g(x, y, z), \tag{10}$$

where $g(x, y, z)$ is the pressure on boundary Γ_1, MPa.

The second kind of boundary of the Eq. (9) is

$$\boldsymbol{v}\cdot n|_{\Gamma_2} = q(x, y, z), \tag{11}$$

where $\boldsymbol{n}$ is the direction vector of exterior normal on boundary Γ_2; $q(x, y, z)$ is the flow rate of unit area on boundary Γ_2, μm/s.

Based on the Galerkin principle and use penalty function method [5], which subject to the boundary condition Eq. (10) and Eq. (11). So the meshless Galerkin numerical linear equations is

$$\boldsymbol{Kp} = \boldsymbol{F}, \tag{12}$$

where

$$\boldsymbol{K} = \frac{k}{\mu}\int_{\Omega}\left(\boldsymbol{N}_x^T\boldsymbol{N}_x + \boldsymbol{N}_y^T\boldsymbol{N}_y + \boldsymbol{N}_y^T\boldsymbol{N}_y\right)d\Omega, \tag{13}$$

$$\boldsymbol{F} = -\int_{\Gamma_2}\boldsymbol{N}^T q d\Gamma, \tag{14}$$

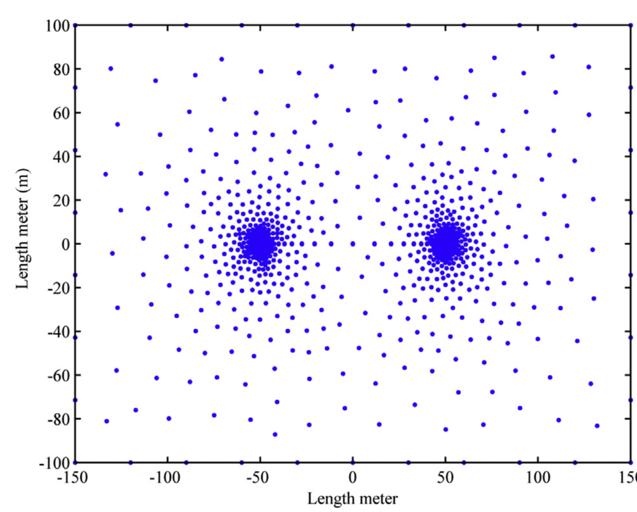

Fig. 2. Nodes distribution of horizontal section.

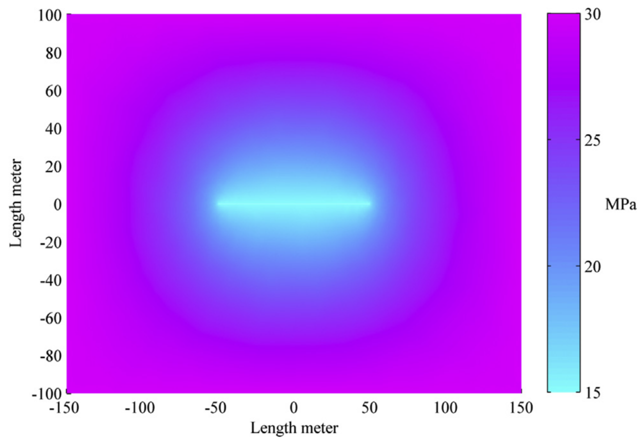

Fig. 3. Pressure distribution of horizontal section.

where $\boldsymbol{N}$ is shape function, $\boldsymbol{N}_x$, $\boldsymbol{N}_y$, $\boldsymbol{N}_z$ are the derivatives of the shape function about x, y, z respectively.

3. Example

In the pictures, Fig. 1 is the vertical section diagram of a horizontal well (2a is the length of reservoir, b is the half width of reservoir, h/2 is the half thickness of the reservoir, 2 l is the length of the horizontal wellbore), the reservoir is a cuboid, with width of 200 m and length of 300 m, thickness 15 m, permeability 25mD, crude oil viscosity 25 mPa s, initial formation pressure 30Mpa, flowing bottom hole pressure 15Mpa, length of horizontal well 100 m, and inner diameter of wellbore 0.1 m.

Stationing in the solution domain is based on the sparse degree of streamline, the nodes distribution on the horizontal section along the axis of the horizontal wellbore is shown in Fig. 2.

Near the two sides of the wellbore, nodes distribution is radial. It insures the calculation accuracy. Use program to simulate the horizontal well reservoir, and cut off the pressure distribution on the horizontal section as shown in Fig. 3.

The pressure distribution of the near wellbore is approximate to ellipse shape. It is also similar to the pressure distribution of the vertical fractured well reservoir in constant pressure production condition.

4. Conclusions

This paper applies meshless Galerkin to simulation of the pressure distribution of horizontal well reservoir, and conducts simulation calculation. Meshless Galerkin is a feasible numerical method for the horizontal well reservoir simulation. Furthermore, meshless Galerkin can be applied to the simulation of multilateral wells, vertical fractured wells and multistage fractured wells. So it is useful to research these complex reservoirs.

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